Maths Vision Statement – To encourage our children to become lifelong mathematicians by equipping them with a set of key skills and developing an inquisitive mindset that inspires them to be the best that they can be.

This policy contains the key mental and practical, pencil-paper procedures that are to be taught: it has been written to ensure consistency and progression throughout the school in addition, subtraction, multiplication and division.

This document highlights and demonstrates the importance of children's conceptual understanding to support the use of mental calculations, leading to the understanding of written calculations when needed (in line with 2014 Primary National Curriculum). Children's understanding of number should allow them to be able to recognise the most efficient method when completing calculations. In order to achieve this, we follow the use of a concrete, pictorial, abstract approach to develop Mastery Maths on which our whole maths curriculum is based upon.
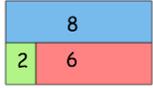
This is underpinned by the use of models and images that support conceptual understanding and this policy promotes a range of representations to be used across the school. Mathematical understanding is developed through use of representations that are first of all concrete (e.g. dienes, counters and multilink cubes), and then pictorial (e.g drawing place value charts, part whole model and bar model) to then facilitate abstract working (e.g. mental calculations and formal written methods).
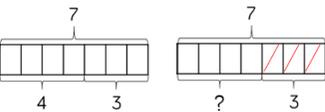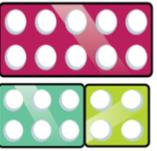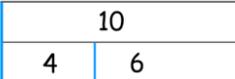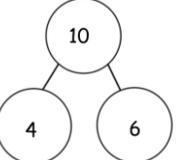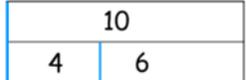
At the centre of the Mastery approach to the teaching of maths is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their conceptual understanding of these procedures through the use of concrete materials, pictorial representations and use of explanations using the correct mathematical language.

It is important to recognise that the ability to calculate mentally lies at the heart of maths. Understanding of place value, fluency in mental methods and good recall of number facts, such as multiplication tables and number bonds and recognising relationships between numbers are vital steps before formal written methods are taught.

Within this document, there is an overview of the expectations for each key stage, a breakdown of the calculation objectives taught for addition, subtraction, multiplication and division, key vocabulary and a glossary of terminology relating to Mastery Maths. Each calculation objective is then broken down into concrete, pictorial and abstract representations for each year group.

Addition			
Objective	Concrete	Pictorial	Abstract
Year 1

Additon of numbers under 10. | **Introducing Part Whole**
Children to use figures and hoops to create part whole models.

Children to use the hoops and numicon/counters.

Cuisenaire rods.

Use of multilink and drawing on the table to show the part and whole. | **Drawing Part Whole Models**
Children to draw representation to show understanding of part and whole and their relationship.
Children to use sentence stems to be able to say what their pictorials show.
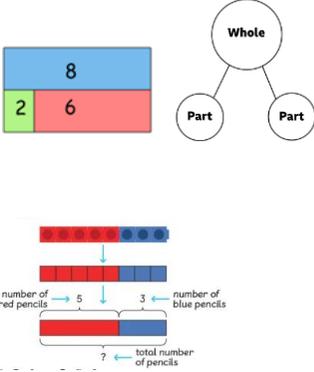
 | **Writing Part Whole Number Sentences**
Children to write the calculation alongside the whole part models.
Children to start to develop a meaning of the equal sign (not answer). Use of sentence stems.

6 + 2 = 8 six add two is the same as eight.
8 = 2 + 6 eight is the same as (equivalent to) six add two. |

		Concrete	
Number bonds to 10 and 20	Representing Number Bonds to 10 and 20 Use of: Numicon Multilink Cubes Tens frames and two-sided counters (yellow and red are available in school)	Drawing Number Bonds to 10 and 20 Bar Model | 10	
4	6 | Part Whole Model 6 parts add 4 parts the whole 10. Tens Frame	Oracy and Written Number Bonds to 10 and 20 | 10	
4	6	 10 = 6 + 4. Ten is the same as six add four 10 = 4 + 6. Ten is the same as (equivalent to) four add six. 4 + 6 = 10 Four add six is the same as ten. 6 + 4 = 10
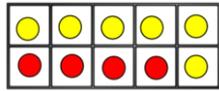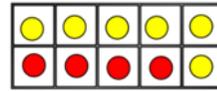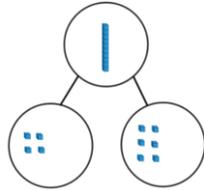
	  Part Whole Model 		Six and four is equivalent to (the same as) ten.
	Representing Number Bonds Within 10 Multilink Cubes: Bead Strings: Part Whole Models with Counters/ Figures: Use of 5 Frames:	**Drawing Number Bonds Within 10** Bar Model: Part Whole Model: Children could draw the counters in the bar model or part whole model and use the sentence stems to say what is being represented.	**Written Number Bonds Within 10** Children to write the calculations and to use sentence stems to read out the number sentence: **3 + 7 = 10** $7 = 3 + 4$ **(Reminder of one digit per square)** Make sure to include examples where one of the parts is zero. Encourage to write their number sentences in different orders and to use show their understanding of the equal to sign (not that it means answer).
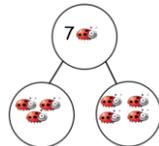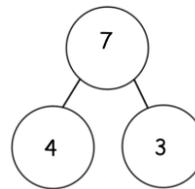
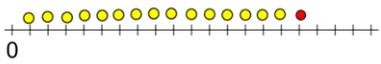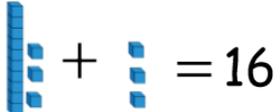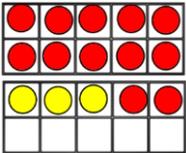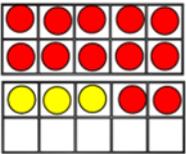|			
Understanding place value in teen number.	**Representing understanding teen numbers as a complete 10 and some more** Tens Frames: Bead Strings (Rekenrek): Part Whole Model (with objects): Base Ten: 	**Drawing understanding teen numbers as a complete 10 and some more** Children can draw tens frames in their books, draw part whole model or base ten. Part-whole 	**Written/ mental understanding teen numbers as complete 10 and some more** 1 ten and 3 ones is equivalent to 13 ones 1 ten and 3 ones is equal to 13 10 + 3 = 13
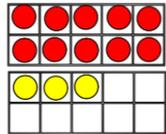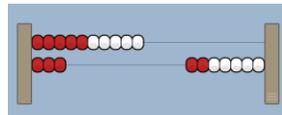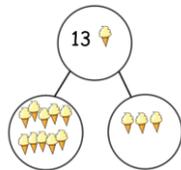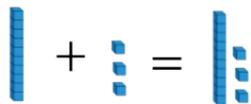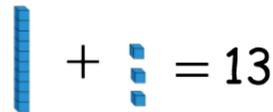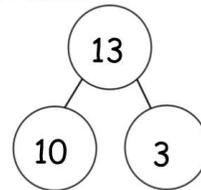
	Numicon: Sentence Stem There are 10 parts and 3 parts so the whole is 13. 10 parts add 3 more parts makes a whole of 13.		
Counting and adding more.	**Counting and adding more** Use of figures/ counters. **Children to use sentence stems to support them:** 1 more than fifteen is the same as…. Fifteen add one more is equivalent to…… Use of whiteboard number lines using objects or counters: 	**Counting and adding more** Children to draw base ten in their books: Children to draw number lines: 	**Counting and adding more** 3 more than 15 is 18 3 + 15 = 18 15 add 3 more is the same as 18 15 + 3 = 18 Children to start to create their own stories: Miss C has 5 pennies but I have 6 more than Miss C. I have 11 pennies.

	Encouragement of children writing the numbers on the number line with a support map. Children to develop the understanding of zero. Encourage children to link their knowledge of number bonds within 10. Use of multilink cubes and base ten.		
Adding the ones.	Adding the ones Children use a range of manipulatives to recognise how to add the 1s to find the total efficiently. Bead strings: Tens frames: Base 10:	Adding the ones Children to draw the tens frame of the base 10 in their books to support their understanding. Children can do this alongside the number sentence to support their understanding.	Adding the ones Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently. Children can verbally explain how they are adding in their heads: I know 2 add 3 is the same as 5 and I have 1 ten so I have 15.... 12 + 3 = 10 + 5 = 15

	2 + 3 = 5 12 + 3 = 15		
Addition by using knowledge of number bonds to bridge the 10.	Addition by using knowledge of number bonds to bridge the 10. Must include the understanding of zero. Use of bead strings: Tens frames: Base 10: Numicon (placing the ten on top):	Addition by using knowledge of number bonds to bridge the 10. Children to draw the tens frame in their books to support them: Children could also use a number line to support them:	Addition by using knowledge of number bonds to bridge the 10. Children able to verbalise their understanding: I know 7 add 3 equals 10 and then I add 2 more.
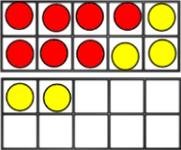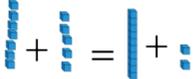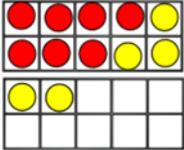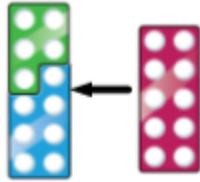
7 + 5 = 13

7 add 3 is the same as 10.
So, 7 add 5 is 10 and 2 more.

	Concrete	Pictorial	Abstract
Year 2 Addition Counting on in tens. Children to have a strong understanding that addition is cumulative so can be done in any order. Counting on in 2s, 3s and 5s from 0 (this will help	**Counting on in tens** Children to have a strong understanding of place value of 2 digit numbers and partitioning with the use of the part whole model (the use of the term value is extremely key). **Use of Base Ten/ Introduction of Place Value Counters (linked to place value):** Tens Ones $22 + 10$ **Use of 100 square with counters:**	**Counting on in tens** Children drawing out the place value grid and counters alongside the calculation to support their understanding. Tens Ones 10 10 10 1 1 Use of the bar model to further develop their understanding of more: Miss L has 10 more pennies than Miss M. Miss M has 26 pennies.	**Counting on in tens** Children able to verbalise their understanding of ten more and the relationship with place value. | Number	10 more
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22	32	
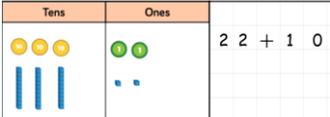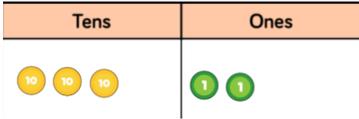
children to later make the link with repeated addition and multiplication).		Miss M [26] Miss L [26][10]	
Children to be secure with mental calculations linked to number bonds to 10 that don't bridge the ten: Eg: 35 + 5 = 42 + 8 =	Addition of 2 digit numbers with ones (bridging the ten). Tens frames 	Addition of 2 digit numbers with ones (bridging the ten). Children can also use the number line to support them: 26 27 28 29 30 31 32 33	Addition of 2 digit numbers with ones (bridging the ten). Children able to verbalise their understanding: 26 add 7 is equal to 33 because 26 add 4 is 30 and then I have to add 3 more.
	Adding 2 digit numbers with ones (adding the ones) Children should be secure of the knowledge of number bonds to 10.	Adding 2 digit numbers with ones (adding the ones)	Adding 2 digit numbers with ones (adding the ones) Children able to verbalise their understanding:
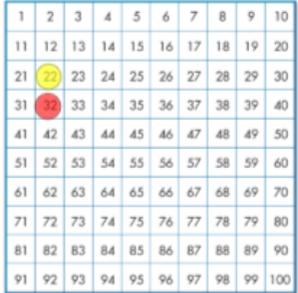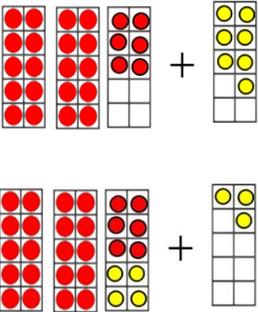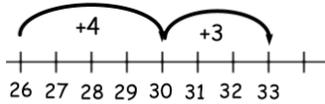
	Children use their knowledge of number bonds to add the ones – understanding the use of term 'exchange'. Use of base ten/place value counters: **26 + 7 =**	Children to draw the place value grid and counters or base ten alongside the number sentence: Children to verbalise what they are doing it as they are doing it (links to place value): 6 ones add 7 ones is 13 ones. 20 and 13 ones is 33 ones I know this as I have exchanged 10 ones to 1 more ten.	26 + 7 = 20 + 13 = 33
Children to start with building the understanding of adding tens by using the relationship with the knowledge of	Addition of 2 digit numbers with 2 digit numbers (starting with understanding of just adding multiples of 10) Use of place value counters/ base ten:	Addition of 2 digit numbers with 2 digit numbers (starting with understanding of just adding multiples of 10) Children to draw the place value grid and counters alongside the calculation to support understanding:	Addition of 2 digit numbers with 2 digit numbers (starting with understanding of just adding multiples of 10) Children able to verbalise their understanding without the use of pictorials (links to place value):
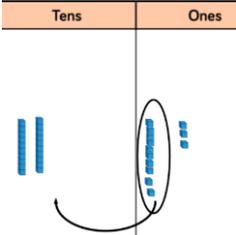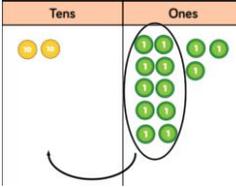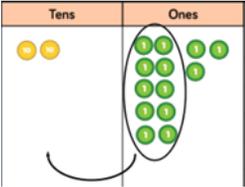
			26 + 50 = 76
number bonds with 10 (links to place value): Eg 20 + 30 = 2 + 3 = 5 so I know 20 + 30 = 50 2 tens add 3 tens is equal to 5 tens	Use of hundred square with counters to support (links to counting on in 10s): Sentence Stems: I have 2 tens and 6 ones. If I add 5 more tens then I have 7 tens and 6 ones. I have 76.		
Addition of 2 digit numbers with 2 digit numbers (no exchanging)	Addition of 2 digit numbers with 2 digit numbers (no exchanging) Children to start with using place value counters and base ten to add the ones and the tens	Addition of 2 digit numbers with 2 digit numbers (no exchanging) Children to draw the place value grid or base ten alongside the	Addition of 2 digit numbers with 2 digit numbers (no exchanging)

	separately (understanding of partitioning and recombining place value): 26 + 32 = Children to verbalise their understanding: 2 tens add 3 tens is equal to 5 tens 20 add 30 is equal to 50 6 ones add 2 ones is equal to 8 ones There are 5 tens and 8 ones altogether/ in total. 	calculations to show their understanding: 20 + 30 = 50 6 + 2 = 8 50 + 8 = 58	Children able to complete calculation mentally or without pictorials: 20 + 30 = 50 6 + 2 = 8 50 + 8 = 58 Mental calculation should be encouraged to rally show children's understanding of place value.
Children to start with building the understanding	Addition of 2 digit numbers with 2 digit numbers (with exchanging)	Addition of 2 digit numbers with 2 digit numbers (with exchanging)	Addition of 2 digit numbers with 2 digit numbers (with exchanging)
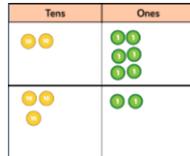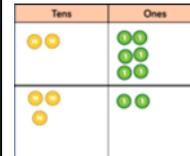
| of adding number bonds to 100 and within 100 using their number bonds to 10 to support:

Eg

45 + 35 =

45 + 55 = | Children to use the place value grid with counters and base ten (set out like the written calculation method):
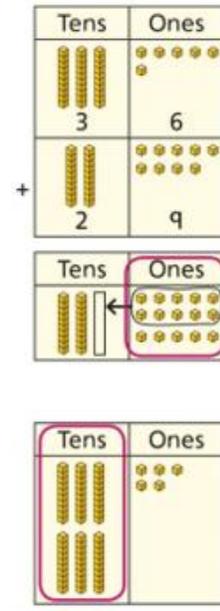
36 + 29 =

6 ones add 9 ones is equal to 15 ones

3 tens add 2 tens is equal to 5 tens and then I add the exchanged ten. In total I have 6 tens and 5 ones | Children to draw the place value grid and counters in their books alongside the written calculation. | Children to understand place value to set out the written method:
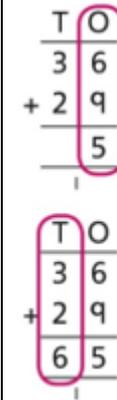
Some children may be able to verbalise how they are adding the two numbers mentally with their understanding of bridging the 10/ exchanging. |

Adding three numbers.	$7 + 6 + 3 = 16$ ⌄ 10	$7 + 6 + 3 = 16$ ⌄ 10 16	$7 + 6 + 3 = 16$

KS2	

	Concrete	Pictorial	Abstract
Year 3 Addition Strong development of using their knowledge of place value to carry out mental calculations	Unitise 100s 10s and 1s to build 3-digit numbers.	100 200 300	Recognise and place numbers on a number line and count backwards and forwards to 1,000.

| before progressing on to the progression of the written methods.

Use of partition to build conceptual understanding before formal written method. |

Understand the links between 1s 10s and 100s.
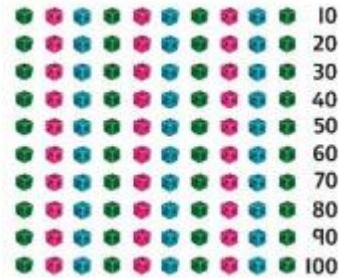
 | Count in steps of 10s and 100s and use equipment and place value grids to represent number up to 1000. |

Use part whole models to represent numbers up to 1,000. |
| --- | --- | --- | --- |
| Adding 3 digit numbers to 3 digit numbers starting with multiples of 100.

add and subtract numbers mentally, including: a three-digit number and 1s, a three-digit number and 10s, a three-digit number and 100s Add with 3-digit numbers Adding a 3-digit number and 1s using models Add and subtract numbers with up to 3 digits, using | 3 hundreds + 5 hundreds = 8 hundreds

$300 + 500 = 800$

Write out the calculation alongside any concrete resources to link it to the written method. | | |

formal written methods of columnar addition and subtraction Formal addition up to 3 digits Adding numbers up to 3 digits (no exchanging) Estimate the answer to a calculation and use inverse operations to check answers Estimate calculations Recognising and using inverse relationship Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction			
3-digit number plus ones with no exchanging.	Use number bonds to add the 1s. 214 + 4 = ? Now there are 4 + 4 ones in total. 4 + 4 = 8 214 + 4 = 218 Base ten 	Part-whole model 	Column addition 214 + 4 ――― 218
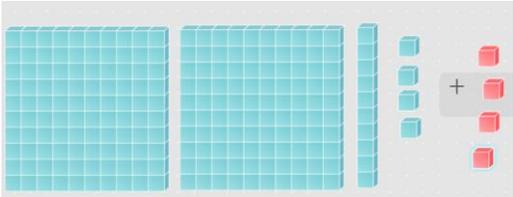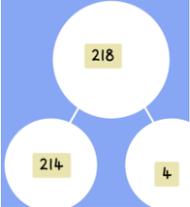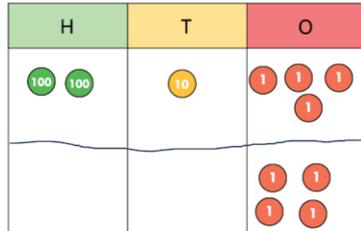
	Place value counters	Bar model	
To add a 3 digit number and tens no exchanging	Use knowledge of number bonds to add the tens. 234 + 50 There are 3 tens and 5 tens altogether. 3 + 5 = 8 In total there are 8 tens. 234 + 50 = 284 Base ten Place value counters and the place value grid	Use knowledge of number bonds to add the tens. 351 + 30 = ? 5 tens + 3 tens = 8 tens 351 + 30 = 381 Bar model Part-whole model	Use knowledge of number bonds to add the tens. 234 + 50 I know that 3 + 5 = 8 So, 30 + 50 =80 234 + 50 = 284 Column addition
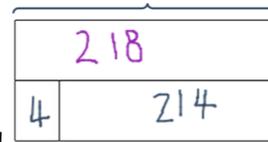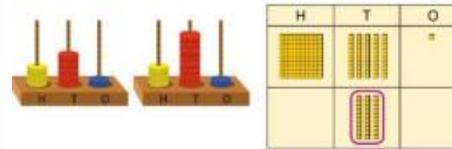
Bar model top: 218 / 4 / 214

	H	T	O		

Place value chart:
- H: 100, 100
- T: 10, 10, 10 (and below: 10, 10, 10, 10, 10)
- O: 1, 1, 1, 1
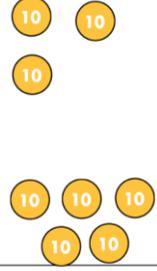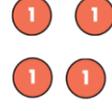
To add a 3 digit number and hundreds no exchanging.	To apply knowledge of number bonds when adding hundreds.	To apply knowledge of number bonds when adding hundreds.	Column method
	3 hundreds + 5 hundreds = 8 hundreds		

To apply knowledge of number bonds when adding hundreds.

$$3 \text{ hundreds} + 5 \text{ hundreds} = 8 \text{ hundreds}$$

$$300 + 500 = 800$$

Write out the calculation alongside any concrete resources to link it to the written method.

$$214 + 100 =$$

700 → 500, 200

314 → 214, 100

Column method

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 H T O
 2 1 4
+1 0 0
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 3 1 4
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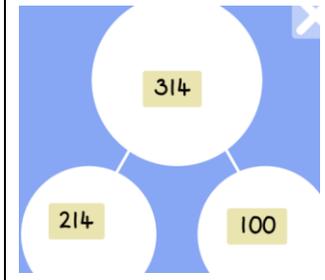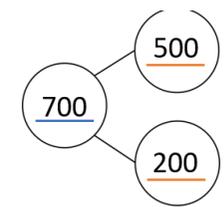
| | | 214 100 | |
		314	
Adding numbers up to 3 digits with exchanging starting with concrete and then the column method.	Use place value equipment to make and combine groups to model addition.	Represent the required exchange on a place value grid using equipment.	Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation.
Estimate the answers to questions and use inverse operations to check answers.	Estimating 98 + 17 = ? 100 + 20 = 120	Use number lines to illustrate estimation.	Building up known facts and using them to illustrate the inverse and to check answers: 98 + 18 = 116 116 − 18 = 98 18 + 98 = 116 116 − 98 = 18
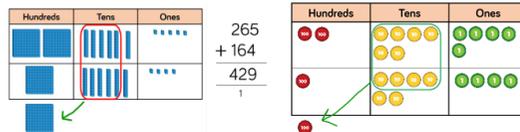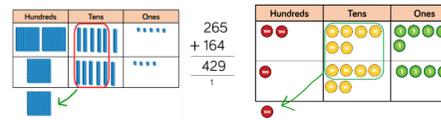
Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. 275 + 99 = ? 275 + 99 = 374 	Use representations to support choices of appropriate methods. I will add 100, then subtract 1 to find the solution. 128 + 105 + 83 = ? I need to add three numbers addition problems. 275 + 99 = ? 275 + 99 = 374 128 + 105 = 233
Objective	Concrete	Pictorial	Abstract
Year 4 Strong development of using their knowledge of place value to carry out mental calculations before progressing on to the progression of the written methods.	Add numbers with up to 4 digits Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	Draw representations using place value grid. 	Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
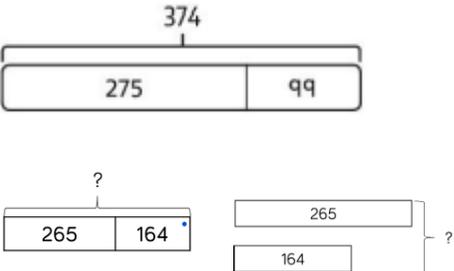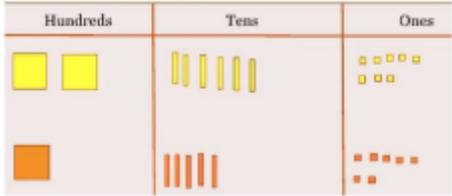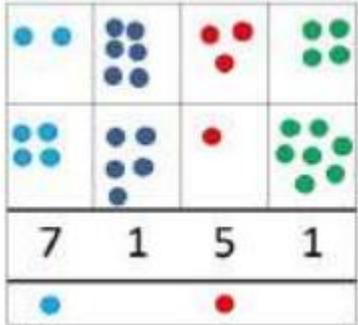
	Concrete	Pictorial	Abstract
Year 5 Addition	Use of place value (and rounding) knowledge to estimate the addition of whole numbers with more than 4 digits. Use of part whole models to partition and discuss estimation: 45 thousand and 23 thousand altogether is 68 thousand. However, I know I then have more as I have 6 hundreds and 4 hundreds. This can be done in a number of ways of partitioning (e.g. just focusing on the total of thousands in both numbers).	Use of place value (and rounding) knowledge to estimate the addition of whole numbers with more than 4 digits. Children to represent their understanding of estimation with the use of part whole models to support them: 	Use of place value (and rounding) knowledge to estimate the addition of whole numbers with more than 4 digits. Children to verbalise their understanding with their knowledge of place value: 45,678 + 23,456 Estimation: 45,000 + 23,000 = 68,000 However, I also know that 600 + 400 is equivalent to 1000 so the answer is going to be slightly greater than 69,000.
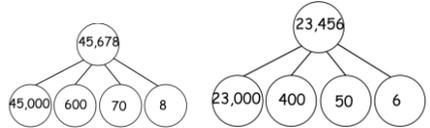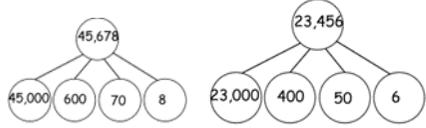
Check children have a secure understanding of mental addition reasoning with their knowledge of place value: Eg: 1200 + 1450 2345 + 435 40,000 + 5,000 + 230 + 40	Addition of whole numbers with more than 4 digits using the formal written method. (Inclusion of the addition of more than two whole numbers). Use place value equipment to represent additions. Use of the place value grid and place value counters to explain understanding (this to be done alongside the written calculation to support understanding): 	Addition of whole numbers with more than 4 digits using the formal written method. (Inclusion of the addition of more than two whole numbers). Represent additions, using place value equipment on a place value grid alongside written methods. I need to exchange 10 tens for a 100. 	Addition of whole numbers with more than 4 digits using the formal written method. (Inclusion of the addition of more than two whole numbers). Use column addition, including exchanges.
	Solving addition multistep problems. Use of the Cuisenaire rods to represent the bar model: 	Solving addition multistep problems. 	Solving addition multistep problems. Use approximation to check whether answers are reasonable.
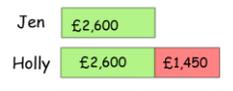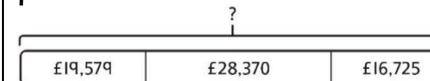
		Jen £2,600 Holly £2,600 £1,450 ? £4,050
		
		 Th H T O
		 2 6 0 0
		 + 1 4 5 0
		 4 0 5 0
		
		 Th H T O
		 2 6 0 0
		 + 4 0 5 0
		 6 6 5 0
			TTh Th H T O TTh Th H
			    2 3 4 0 5 2 3 4
			 + 7 8 9 2 + 7 8
			    2 0 2 9 7 3 1 2
			
			 I will use 23,000 + 8,000 to check. Reasoning logically when the answer cannot be correct.

	Adding tenths (use of place value knowledge to support reasoning and mental calculation). Link measure with addition of decimals. Two lengths of fencing are 0·6 m and 0·2 m. What is the length altogether? 0·6 m 0·2 m Use of place value counters. 	T	O	Tth	 Use of base ten (cube represents a tenth):	**Adding tenths (use of place value knowledge to support reasoning and mental calculation).** Use a bar model with a number line to add tenths: 0·6 m 0·2 m 0·1 m 0·1 m 0·1 m 0·1 m 0·1 m 0·1 m 0·1 m 0·1 m 0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1 0·6 + 0·2 = 0·8 6 tenths + 2 tenths = 8 tenths Exchanging (understanding of a whole): 0.8 + 0.7	**Adding tenths (use of place value knowledge to support reasoning and mental calculation).** Understand the value to support addition: $\dfrac{6}{10} + \dfrac{2}{10} = \dfrac{8}{10}$ 6 tenths + 2 tenths = 8 tenths 0·6 + 0·2 = 0·8 Exchanging (understanding of a whole): 0.8 + 0.7
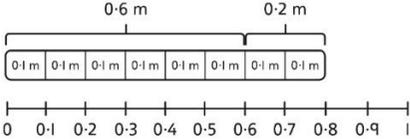
One Whole (1.0 or $\frac{10}{10}$)

| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |

0.5 or $\frac{5}{10}$

| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | | | | | |

$$\frac{8}{10} + \frac{7}{10} = \frac{15}{10} = 1.5$$

8 tenths + 7 tenths = 15 tenths = 1 whole and 5 tenths

0.8 + 0.7 =1.5

(Children then using their conceptual understanding to recognise that 5 tenths is equivalent to a half)

Bead string and rekenrek:

Exchanging (understanding of a whole):

0.8 + 0.7

T	O	Tth

	T	O	Tth
		①	
		●	0.1 0.1 0.1 0.1 0.1
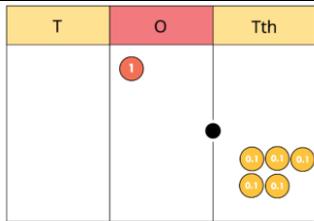
Also done with base ten and tens frame and bead string.

| Children to develop their mental calculation recognising the link with number bonds to 100 and number bonds to a whole. Writing the questions as fractions to support their reasoning:

E.g:

0.65 + 0.35 =

0.52 +   = 1 | **Adding hundredths.**

Linking measure and money with addition of decimals

65p + 82p

£0.65 + £0.82
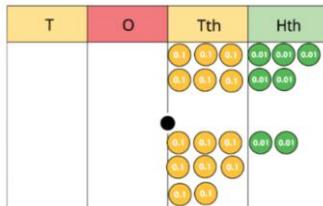
 | **Adding hundredths.**

Use of the hundredths square or place value grid drawn in books.

$\frac{33}{100} + \frac{55}{100}$ | **Adding hundredths.**

Understand the value to support addition:

$\frac{65}{100} + \frac{82}{100} = \frac{147}{100} = £1.47$ |
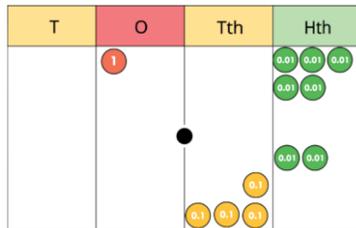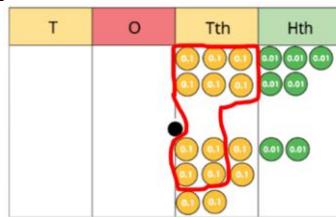
	  Bead strings and rekenreks to represent having a whole. Use of base ten.			
	Adding decimals using the formal written method. Use place value equipment (counters and base ten) to represent additions alongside the written method to support understanding:	**Adding decimals using the formal written method.** Use place value equipment on a place value grid to represent additions. 	**Adding decimals using the formal written method.** Add using a column method, ensuring that children understand the link with place value.	
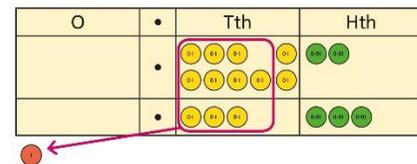
	O	•	Tth	Hth
		•	●●●	●●
		•	●●●●●	●●
		•	●●●	●●●

$$\begin{array}{r} 0 \cdot 9\ 2 \\ +\ 0 \cdot 3\ 3 \\ \hline 1 \cdot 2\ 5 \\ \hline {}_1 \end{array}$$

Include examples where the numbers of decimal places are different.

$$\begin{array}{r} 0 \cdot 9\ 2 \\ +\ 0 \cdot 3\ 3 \\ \hline 1 \cdot 2\ 5 \\ \hline {}_1 \end{array}$$

Include additions where the numbers of decimal places are different.

33.23 + 8.91

	Concrete	Pictorial	Abstract
Year 6 Addition	Selecting mental methods for larger numbers where appropriate Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods. [place value grid: M ●● / HTh ●●●● / TTh ● / Th ● / H ●●● / T / O ●] 2,411,301 + 500,000 = ? This would be 5 more counters in the HTh place. So, the total is 2,911,301. 2,411,301 + 500,000 = 2,911,301	Selecting mental methods for larger numbers where appropriate. Use a bar model to support thinking in addition problems. 257,000 + 99,000 = ? [bar model: ? over £257,000	£100,000] I added 100 thousands then subtracted 1 thousand. 257 thousands + 100 thousands = 357 thousands 257,000 + 100,000 = 357,000 357,000 – 1,000 = 356,000	Selecting mental methods for larger numbers where appropriate. Use place value and unitising to support mental calculations with larger numbers. 195,000 + 6,000 = ? 195 + 5 + 1 = 201 195 thousands + 6 thousands = 201 thousands So, 195,000 + 6,000 = 201,000

		So, 257,000 + 99,000 = 356,000	
	Comparing and selecting efficient methods Children should be demonstrating a secure understanding of place value and addition at this stage.	**Comparing and selecting efficient methods** Discuss similarities and differences between methods and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations. Use bar model and number line representations to model addition in problem-solving and measure contexts. 	**Comparing and selecting efficient methods** Use column addition where mental methods are not efficient. Recognise common errors with column addition. *32,145 + 4,302 = ?* *Which method has been completed accurately?* *What mistake has been made?* Column methods are also used for decimal additions where mental methods are not efficient.
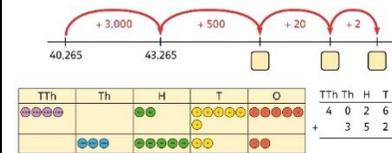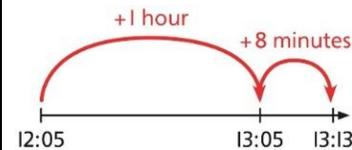
Subtraction			
	Concrete	Pictorial	Abstract
Year 1 Subtraction Focus first on using knowledge of number bonds to represent and find missing parts before building to random wholes within 20. Use of concrete and pictorial to explore the different ways the number sentence could be written (relationship between addition and subtraction) Lots of reasoning and problem solving to be done with concrete objects to develop understanding and language.	Finding a part There should be 5 people/counters altogether. We have 3 so how many more should we have? Tens Frame: Numicon:	Finding a part Drawing part whole models, tens frames or bar models to support their understanding of applying their knowledge of number bonds to support subtraction. 10 6 ? Concrete Discrete 7 7 4 3 ? 3 Combination 7 4 Continuous ? 7 4 3 ? 3	If the whole is _____ and _____ is a part, then the other part is _____ _____ plus _____ is _____ The bond to _____ for _____ is _____ _____ is a part, _____ is a part and _____ is the whole. $10 = 6 +$ ☐ ☐ $+ 6 = 10$ $10 - 6 = 4$ so $10 - 4 = 6$
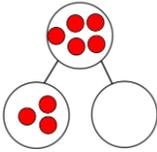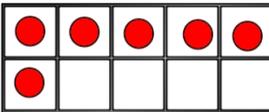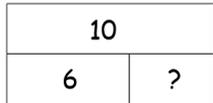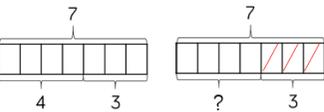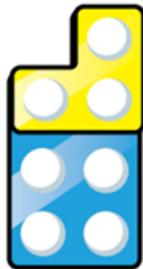
	7 − 3 = 4 If I place a 6 block on top of the 10, how many more are needed?		
Once able to state the eight facts for a calculation, children to then use CPA to answer missing box type questions: 7 = ☐ - 9 Recapping work on addition	**Fact Families – the Eight Facts** Objects to be laid out for children to create fact families (this could include the use of part whole models, number frames, numicon etc)	**Fact Families – the Eight Facts** Children to represent the eight facts through the use of bar models of part whole models Use of the bar model to help state the fact families.	**Fact Families – the Eight Facts** Children to be able to write and verbally state the eight facts. 6 = 2 + 3 6 = 3 + 2 3 + 2 = 6 2 + 3 = 6 6 − 3 = 2 6 − 2 = 3 2 = 6 − 3
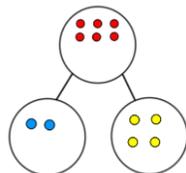
Use of bar model and part whole models to support understanding of the number sentence: ? (part-whole model with yellow and red counters)	Children to talk through or use the counters to show the eight facts from the calculation. (image of green and orange cubes)		3 = 6 – 2
Links to counting backwards on a number line:	**Understanding Subtraction as Taking Away** Children arrange objects and remove to calculate how many are left. Create number stories: Child A has 6 teddies and then she gives 2 to their friend. She now has 4 left. 🧸🧸🧸🧸🧸🧸 (part-whole model: 6 at top, 4 and 2 below) Use of bead strings:	**Understanding Subtraction as Taking Away** Children to draw and cross out pictures to support understanding (encourage the use of the part whole model alongside to show the link): 🔴🔴🔴🔴🔴⊗⊗⊗ Use of a number line (counting backwards): (number line 0–10)	**Understanding Subtraction as Taking Away** Children able to verbally show their understanding and write number sentences. Children able to count backwards.
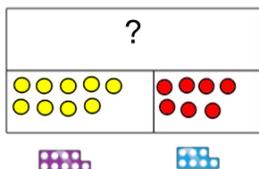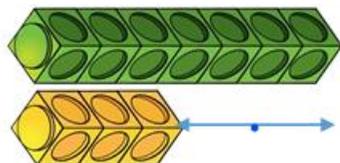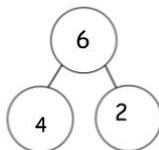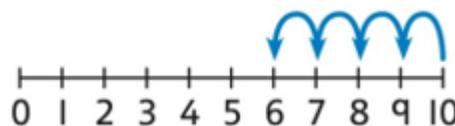
	Finding the Difference Using groups of objects to create stories and develop the language more than, less than and fewer: The are 3 fewer/ less red than yellow. There are 3 more yellow than red. The difference between the red and yellow is 3. Child A has 3 less than me.	**Finding the Difference** Use of drawings of counters or bar model to support understanding: Use of the number line to show understanding: The difference between 10 and 4 is 6. 10 subtract 4 is 6.	**Finding the Difference** Children able to verbalise their understanding of finding the difference and the link with subtraction. The difference between 10 and 4 is 6. 10 subtract 4 is 6.
Recap of number bonds to 20.	**Subtraction Within 20** **Use of number bond knowledge to bridge the 10:**	**Subtraction Within 20** Representation by drawing tens frames to support them:	**Subtraction Within 20** Children to verbally show their understanding of using their
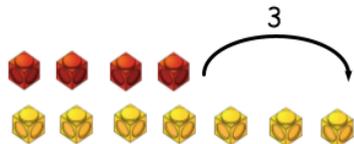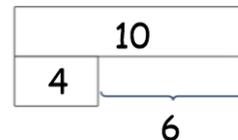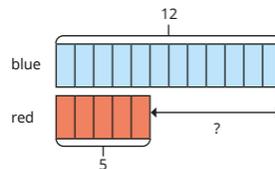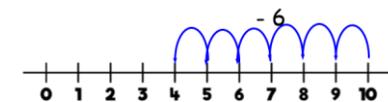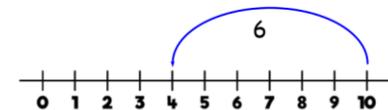
| Do we want subtraction of 10s. | 13 – 7 = 13 subtract 7 The whole 7 is made of 3 parts and 4 parts so I can take away the 3 and then the 4. | Use of the number line: Use of the part whole model: Use of the bar model | knowledge of number bonds to bridge the 10: 13 – 7 = 13 – 3 = 10 10 – 4 = 6 (Remind them that the equal sign means the same as so it can't be lined up in one line e.g. 13 – 7 = 13 – 3 = 10 – 4 = 6) |
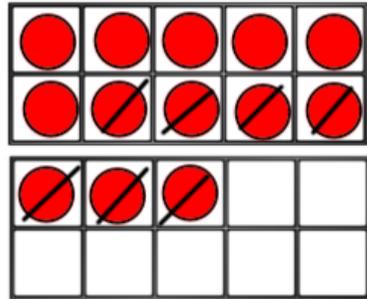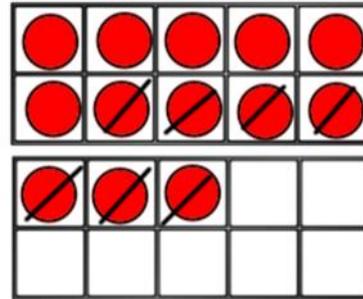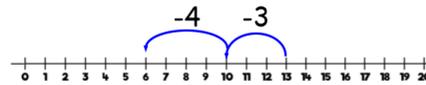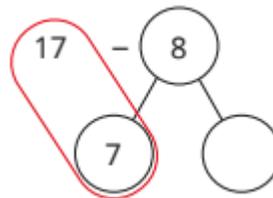
	Concrete	Pictorial	Abstract
Year 2 Subtraction Spend time recapping and reinforcing: children to have a secure understanding and recall of the addition and subtraction facts to 20. Links to be made with counting back in 10s. Children to be taught that subtraction, unlike addition, cannot. Children to read the subtraction questions out (children can still find this very	**Subtracting Multiples of 10** Children to start with building the understanding of subtracting tens by using the relationship with the knowledge of number bonds with 10 (links to place value): 50 - 30 = 5 - 3 = 2 so I know 5 tens subtract 3 tens equals 2 tens 50 – 30 = 20	**Subtracting Multiples of 10** Drawing representations through the use of the bar model and part whole model: 10 tens subtract 3 tens is equal to 7 tens. 	**Subtracting Multiples of 10** 80 – 60 = 8 – 6 = 2 8 tens subtract 6 tens is equal to 2 tens 80 – 60 = 20

challenging in Year 6).	2 tens add 3 tens is equal to 5 tens		
	Subtracting the ones (no bridging of the ten) Use of place value counters and base ten: 56 – 4 Children to use place value language to discuss the values of their tens column and ones column. Children to physically subtract the 4 ones.	**Subtracting the Ones (no bridging of the ten)** Children to draw the place value grid alongside the calculation to support their understanding. 56 – 4 = 5 tens and 2 ones = 52 Use of a number line (this can also be concrete):	**Subtracting the Ones (no bridging of the ten)** 56 is 5 tens and 6 ones 5 tens and 6 ones subtract 4 ones is equivalent to 5 tens and 2 ones. 56 – 4 = 52
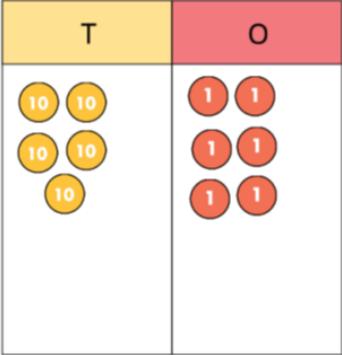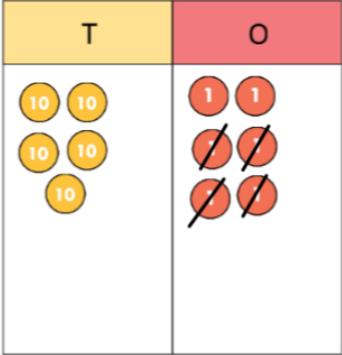
		- 4 52 56	
	Subtracting one digit number bridging the ten Using knowledge of number bonds 43 – 8 Recap understanding of partition (43 is 4 tens and 3 ones) Tens Frame: 43 - ⑧ ③ ⑤ 43 – 3 = 40 40 – 5 = 45	**Subtracting one digit number bridging the ten** Drawing out the tens frame alongside the number sentence to support their understanding: Number line: (Number lines can also be concrete with physically showing what they are doing)	**Subtracting one digit number bridging the ten** 43 – 8 = 35 43 – 3 = 40 40 – 5 = 45
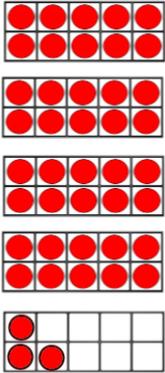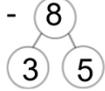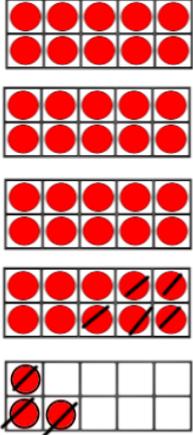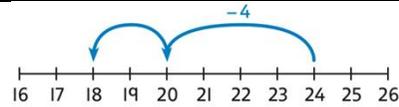
	Subtracting a 2-digit number from a 2-digit number (no bridging)	Subtracting a 2-digit number from a 2-digit number (no bridging)	Subtracting a 2-digit number from a 2-digit number (no bridging)
Encourage children to develop the mental calculation through their understanding of place value. Bar models reinforced to represent a subtraction problem: 	57 – 24 Use of place value grid and place value counters/ base ten (children to try with both): Children to use the correct place value language.	Children to draw out the place value grid and counters alongside the calculation to support their understanding: Children apply their understanding of partitioning and subtraction of multiplies of 10: 57 – 24 57 subtract 2 tens is equal to 37 and then subtract 4 ones.	Children showing a secure knowledge of completing the calculations mentally and able to explain their process using the correct language.
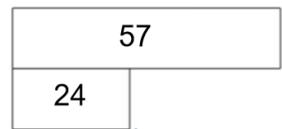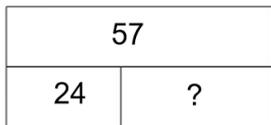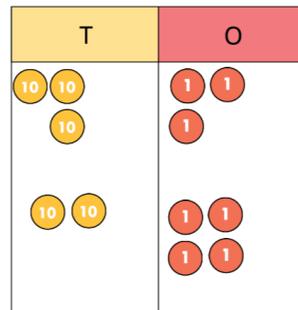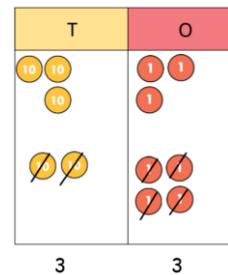
Expanded method to support children's understanding before moving on to the compact method.	Subtracting a 2 digit-number from a 2-digit number involving exchanging.	Subtracting a 2 digit-number from a 2-digit number involving exchanging.	Subtracting a 2 digit-number from a 2-digit number involving exchanging.

Expanded method to support children's understanding before moving on to the compact method.

$$\begin{array}{r} 30 \\ \overset{1}{4\!\!/0} + {}^1 5 \\ - 20 + 7 \\ \hline \end{array}$$

Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s.

Subtracting a 2 digit-number from a 2-digit number involving exchanging.

Use of place value grid and place value counters/ base ten (children to try with both):

57 – 39 (some children may start to recognise that they can subtract 40 and add 1)
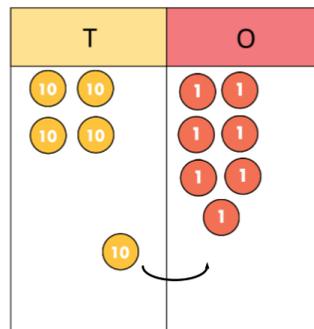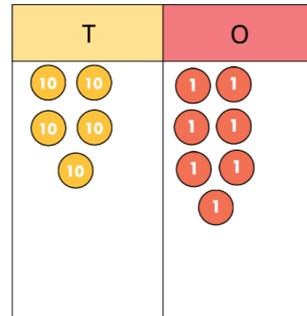
Subtracting a 2 digit-number from a 2-digit number involving exchanging.

Children to draw out the place value grid and counters alongside the formal written method to support their understanding:

Subtracting a 2 digit-number from a 2-digit number involving exchanging.

Children to use the formal written method when necessary. Expanded developing on to the compact:

$$\begin{array}{r} 30 \\ \overset{1}{4\!\!/0} + {}^1 5 \\ - 20 + 7 \\ \hline \end{array}$$

Build to formal written method being completed alongside the place value grid as they work through the calculation.

	Concrete	Pictorial	Abstract
Year 3 Subtraction			
Subtracting 100s 10s and 1s no exchange	Use known facts and unitising to subtract multiples of 1, 10 and 100.	Use known facts and unitising to subtract multiples of 1, 10 and 100.	Understand the link with counting back in 1s, 10s and 100s. 400 – 200 = 200
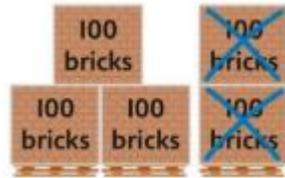
	5 – 2 = 3 50 – 20 = 30 500 – 200 = 300	4 – 2 = 2 40-20 = 20 400 – 200 = 200	400 – 200 = 200 Use known facts and unitising as efficient and accurate methods. I know that 7 – 4 = 3. 70 – 40 = 30 Therefore, I know that 700 – 400 = 300.
Subtracting 1s across a 10 and 10s across a 100. Column method with regrouping.	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters Start with the ones, can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones. Now I can subtract my ones.	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	Children can start their formal written method by partitioning the number into clear place value columns. Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals.
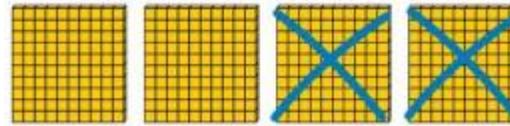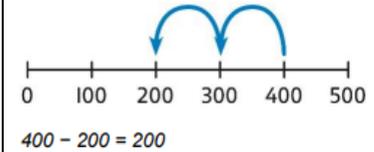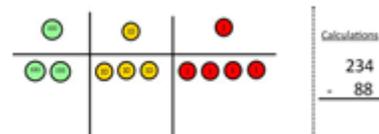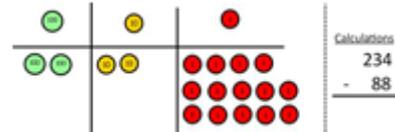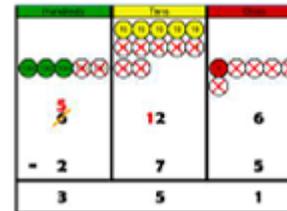
Subtraction of numbers up to 3 digits.	Continue with using place value counters and base ten with column method written alongside.	Use of part whole model and bar models to help with missing number problems. $$435 - 273 = 162$$	Use of column method can now start to be applied to word problems as well. There were 435 students in a school. 273 of them went on a school trip. How many were left at school?
	Concrete	**Pictorial**	**Abstract**
Year 4 Subtraction To subtract up to 4-digit numbers. To also use Year 3 methods for regrouping.	Now look at the tens, can I take away 8 tens easily? I need to exchange 1 hundred for 10 tens. Now I can take away 8 tens and complete my subtraction.	Continue use of part whole model and bar models to support with missing number questions and understanding of subtraction.	Choose appropriate method to use for calculation. E.g. 2300 – 1000 would not necessarily require use of a written method.
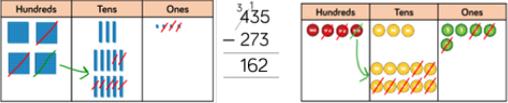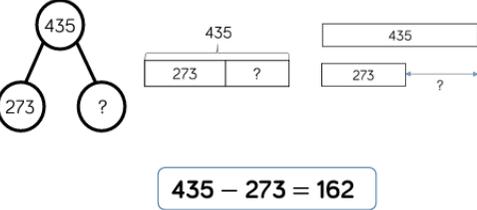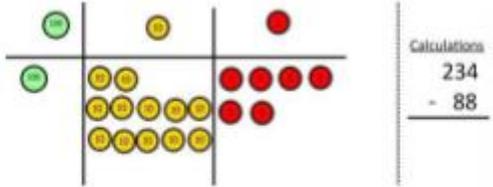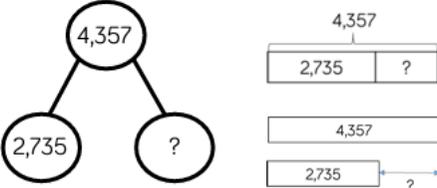
Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

	Concrete	Pictorial	Abstract
Year 5 Subtraction			
Continued use of the bar model and part whole model to support children's understanding.

Continued reinforcement of language (place value, comparative adjectives and difference). | | **Choosing Efficient Subtraction Methods**

50, 000 – 9,000

Using the place value houses/ knowledge to recognise that it is 50 thousands subtract 9 thousands.

Use of multiplication and division facts to support and explain understanding:

250,000 thousand times smaller is 250 and 9000 a thousand times small is equivalent to 9. 250 subtract 9 is 241. 241 a thousand times bigger is 241,000. | **Choosing Efficient Subtraction Methods**

To subtract two large numbers that are close, children find the difference by counting on.
2,002 – 1,995 = ?

Use of rounding:

567 – 99

567 – 100 + 1 = 468

Use of place value:

12,462 – 2,300 = |

Children to be given mixture of question to recap their efficiency with mental calculations.	Subtraction of numbers with more than 4 digits that involve exchanging. Children able to explain in terms of value. Use place value equipment to understand where exchanges are required. 	Subtraction of numbers with more than 4 digits that involve exchanging. Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required. $15,735 - 2,582 = 13,153$ 	Subtraction of numbers with more than 4 digits that involve exchanging. Use column subtraction methods with exchange where required. $62,097 - 18,534 = 43,563$
		Checking strategies and representing subtractions Bar models represent subtractions in problem contexts, including 'find the difference'. 	**Checking strategies and representing subtractions** Children can explain the mistake made when the columns have not been ordered correctly.
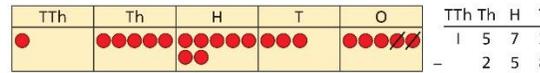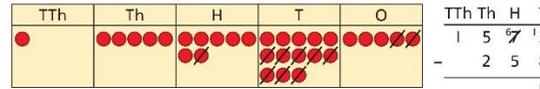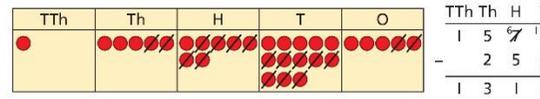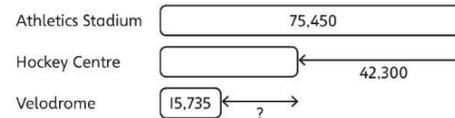
			  Use approximation to check calculations. *I calculated 18,000 + 4,000 mentally to check my subtraction.*
	Subtracting decimals Use of place value counters to explore number bonds to 1. Explore complements to a whole number by working in the context of length and money. I m – ☐ m = ☐ m *1 − 0.49 = ?* $\frac{100}{100} - \frac{49}{100} = \frac{51}{100}$ Use of decimal fraction language and place value counters to support understanding. How many more tenths	**Subtracting decimals** Use a place value grid to represent the stages of column subtraction, including exchanges where required. *5·74 − 2·25 = ?*	**Subtracting decimals** Children should be able to use mental calculations to demonstrate their understanding of value (decimal fractions) and use of number bonds to 1 (linked to number bonds to 10 and 100) when completing basic decimal subtractions. Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. *3·921 − 3·75 = ?*

and how many more hundredths would make a whole? Links to number bonds.

1 is equivalent to a whole so therefore it is equivalent to a hundred hundredths

Links to pence in a pound to value of a whole.

£1.45 – £0.98 =

145p – 98p =

$$\frac{145}{100} - \frac{98}{100} = \frac{47}{100}$$
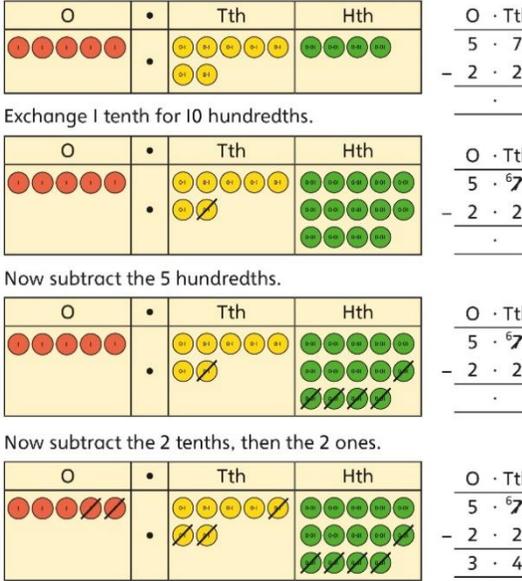
O	.	Tth	Hth

Exchange 1 tenth for 10 hundredths.

Now subtract the 5 hundredths.

Now subtract the 2 tenths, then the 2 ones.

O	.	Tth
5	.	7
– 2	.	2

O	.	Tth	Hth	Thth
3	.	9	2	1
– 3	.	7	5	0

	Concrete	Pictorial	Abstract
Year 6 Subtraction Continued use of the bar model and part whole model to support children's understanding. Continued reinforcement of language (place value, comparative adjectives and difference).			Comparing and selecting efficient methods Reinforcement of Year 5.

Multiplication			
Objective	**Concrete**	**Pictorial**	**Abstract**
Repeated addition Use of language 'lots of' throughout.	 Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there? 2 + 2 + 2 = 6 5 + 5 + 5 = 15 Drawing of arrays to support.	Write addition sentences to describe objects and pictures. 2 + 2 + 2 = 6 3 lots of 2
Counting in 2s, 5s, 10s Pre passport available to support			
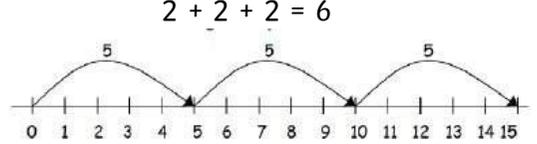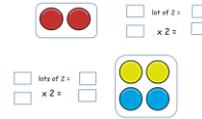
Creating arrays with counters/ making groups of/ lots of

2 Times Tables:
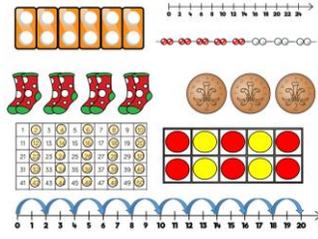
5 Times Tables:

10 Times Tables:

Arrays-showing commutative multiplication	Create arrays using counters/cubes to show multiplication sentences. 	Draw arrays 4 lots of/ groups of 2 = 8 2 lots of/ groups of 4 = 8 Use of the bar model 	Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15
Doubling Starting to recognise the language of doubling linked to 2 groups of	Double 5 2 groups of 5 Use of counters etc to show.	Use of drawing of counters into groups and use of the bar model (which could have drawing of counters on it): 	
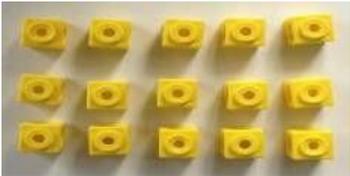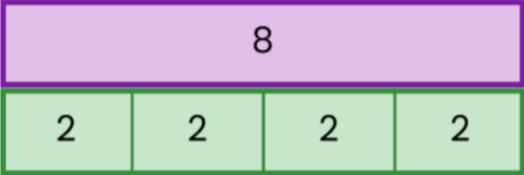
	Concrete	Pictorial	Abstract
Year 2 Counting in multiples Recapping and building on their understanding of counting in 2s, 5s and 10s. Introduction of the multiplication sign	Count the groups, use a range of manipulatives	Children make representations to show counting in multiples Use alongside the written 4 lots of 3, 4 groups of 3 and the introduction of the multiplication sign 4 x 3. Understanding the equal signs to be the equivalent of (the whole).	Count in multiples aloud. Write sequences with multiples of numbers 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30, 35 0, 3, 6, 9, 12, 15
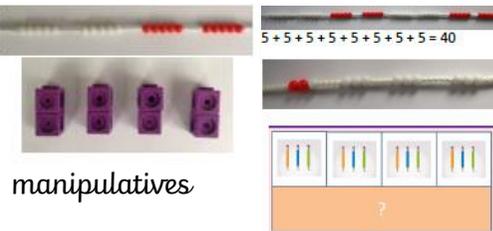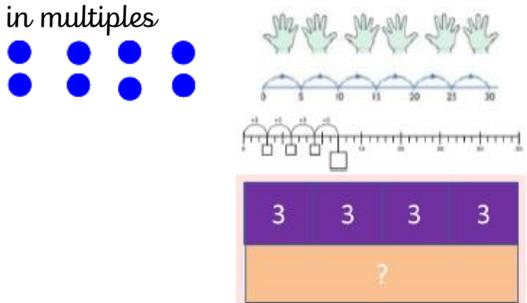
| Making equal groups and counting the total | Use manipulatives to count equal groups | Draw and make representations
Draw to show 2 x 3 = 6
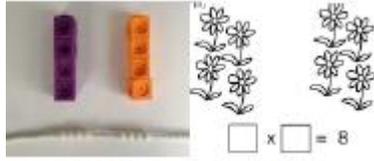
Use of arrays and the bar model.

Draw arrays in different rotations to find
 commutative multiplication sentences.

 4 lots of/ groups of 2 = 8

2 lots of/ groups of 4 = 8

Link arrays to area of rectangles.
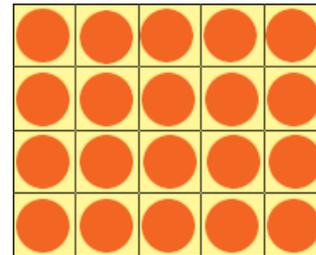
 | 2 x 4 = 8 |

| Using the inverse (this should be taught alongside division, so that pupils learn how they work alongside each other) | | Use of the bar model | 2 x 4 = 8   4 x 2 = 8
8 ÷ 2 = 4   8 ÷ 4 = 2
8 = 2 x 4   8 = 4 x 2
2 = 8 ÷ 4   4 = 8 ÷ 2 |

Objective	Concrete	Pictorial	Abstract
Year 3 3, 4 and 8 times table. Recognising the relationships between the 2, 4 and 8 times table	Use of arrays with counters. Comtinue to develop understanding of cumlative. 3 Times Tables: 4 Times Tables: 8 Times Tables:		

	8 16 24 32 	8	16	24	32	40
48	56	64	72	80 | 0 8 16 24 32 40 48 56 64 72 80 88 96		
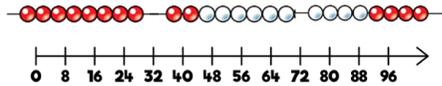
2 digit number multiplied by a 1 digit number with no exchange and the building on to with an exchange.	Show the link with arrays to first introduce the grid method. 4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method. 4 rows of 13	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
7	210	35 | 210 + 35 = 245
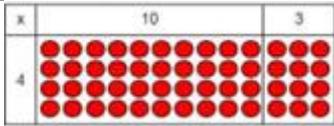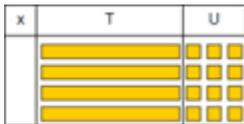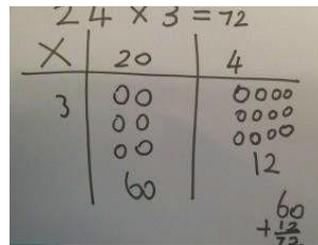
Move on to place value counters to show how we are finding groups of a number. There are two methods. Method 1 builds on to method 2 to help support their conceptual understanding in Year 4.

Method one (linked to multiplication being the same as repeated addition)

13 x 4

13 lots of 4

13 groups of 4
13 + 13 + 13 + 13

Objective	Concrete	Pictorial	Abstract
All multiplication facts up to 12 x 12	Use of arrays with counters. Comtinue to develop understanding of cumlative.		
Year 4 2 digit/ 3 digit number multiplied by a 1 digit number	Same use of stratgies as Year 3 building to the formal written method.		Use of the formal written method

Tens	Ones
10	1 1 1
	1 1 1
	1 1 1
	1 1 1

Tens	Ones
10 10	1 1
10 10	
10	

3 5   3 x 5 = 1 5

X 5   3 tens x 5= 1 5 ten

5

2

H	T	O
2	4	5
×		4
9	8	0
1	2	

Objective	Concrete	Pictorial	Abstract
	$245 \times 4 = 980$		
Multiplying three numbers	Use of counters and knowledge of multiplication facts		$3 \times 2 \times 4$ $3 \times 2 = 6$ $6 \times 4 = 24$

Year 5	Include use of knowledge of place value and rounding to complete estimations first.			Th	H	T	O
					1	8	2	6
Multiplying a 4 digit number by a 1 digit number.			×					3
	1826 x 3				5	4	7	8
						2	1	

2000 x 3 = 6000 (How do they know that the answer will be slighlty less?)
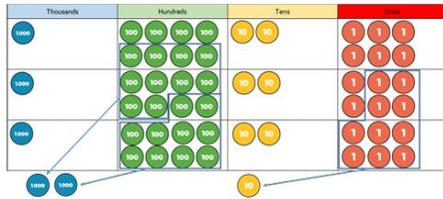
1,826 × 3 = 5,478

Multiplying a 2 digit number by a 2 digit number
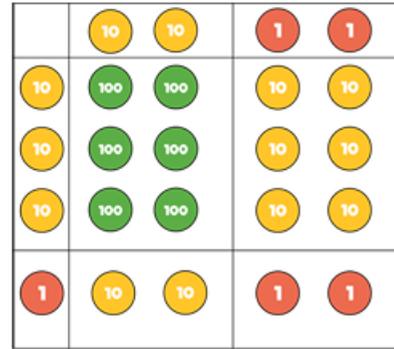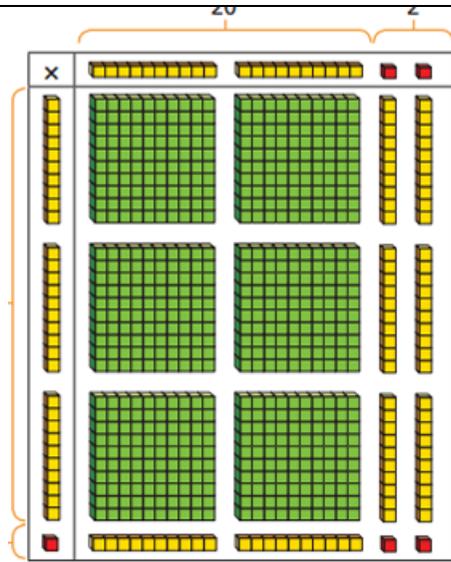
×	20	2
30	600	60
1	20	2

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

Objective	Concrete	Pictorial	Abstract

Multiplying a 3 digit number by a 2 digit number
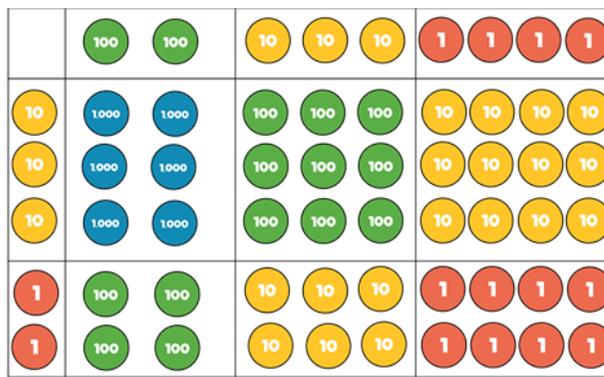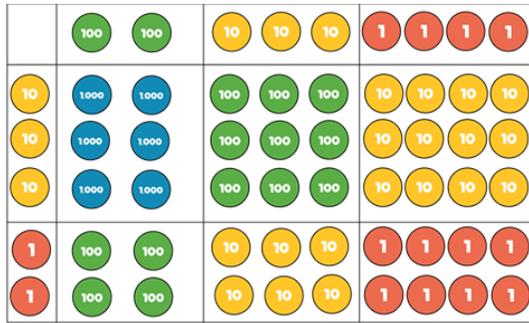
×	200	30	4
30	6,000	900	120
2	400	60	8

Th	H	T	O
		2	3	4
	×		3	2
		4	6	8
1	7 $_1$0	2	0
	7	4	8	8

Year 6	Use of same strategies as Year 5.	Use of same strategies as Year 5.	
4 digit number multiplied by a 2 digit number			

The grid on the right shows:

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 2	1 5	9 3	1 7	2
5 1	4	7 1	8	0
7	6	6	9	2

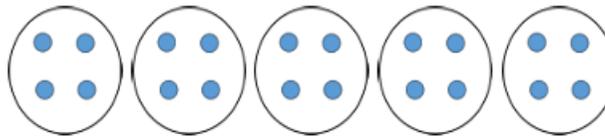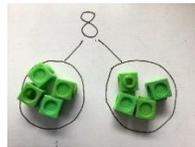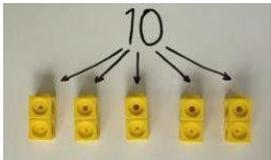| Multiplying decimals up to 2 decimal places by a single digit Still repeating the idea	Use of the place value grid to support understanding: 2.14 x 3 		Remind the children that single digits belong in the ones column. Line up the decimal points in the question and the answer.
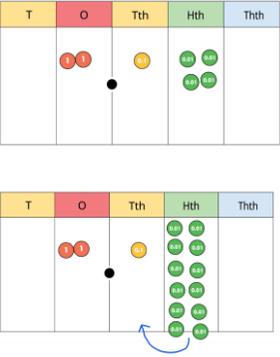
T	O	Tth	Hth	Thth
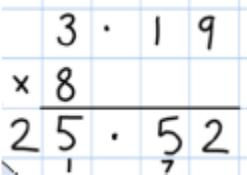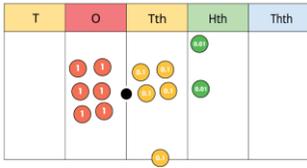
Division

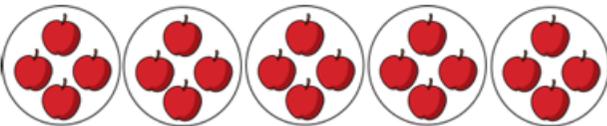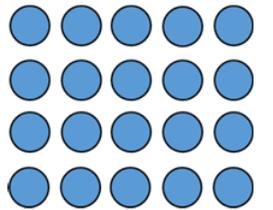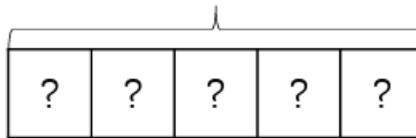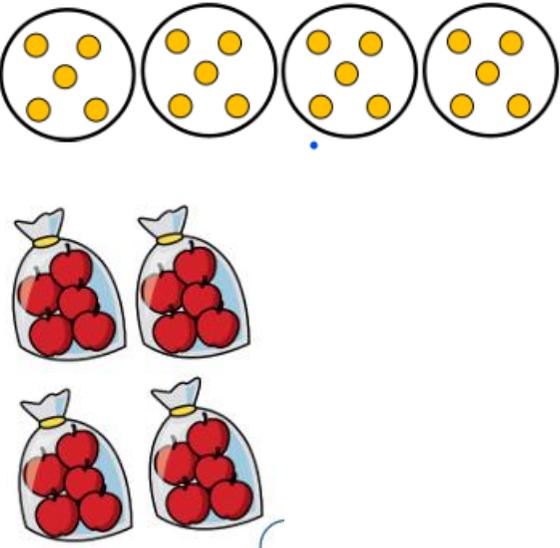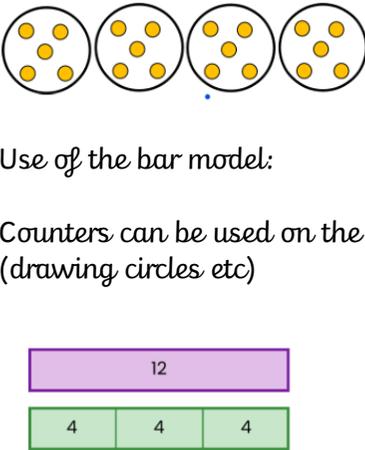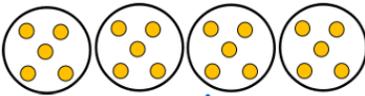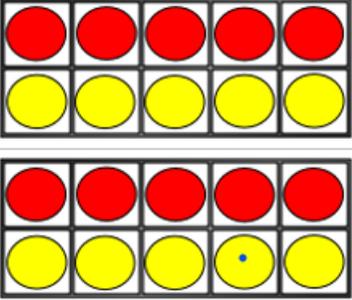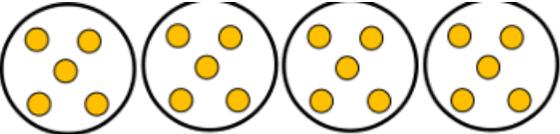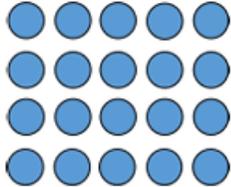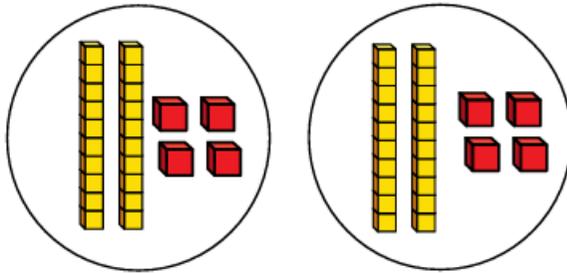| Objective	Concrete	Pictorial	Abstract
Year 1 Understanding division as sharing (Division symbol introduced in Year 2)	Use of concrete objects and counters: 		

Introduction of grouping	Use of concrete objects and counters to support groups of: 	Use of drawing to support knowledge of groups of: Use of the bar model: Counters can be used on the bar model (drawing circles etc) 	12
4	4	4		Writing/ orally saying what they see. There are 4 groups of 5:
Objective	**Concrete**	**Pictorial**	**Abstract**
Year 2 Division facts linked to the 2, 5 and 10 times tables			
Understanding of grouping and sharing as division	There are 20 apples altogether. They are put in bags of 5. How many bags are there?	There are 20 apples altogether. They are put in bags of 5. How many bags are there?	There are 20 apples altogether. They are put in bags of 5. How many bags are there?

| with the use of the division sign. | Use of counters and arrays. | Use of circles to group or share. Use of the bar model. | $20 \div 5 = 4$ |

Objective	Concrete	Pictorial	Abstract
Year 3 Division facts for the 3, 4 and 8 times tables.			

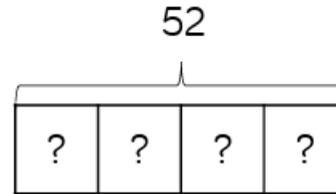| Objective	Concrete	Pictorial	Abstract
Year 3 Divide 2 digits by 1 (sharing with no exchanging)		Drawing of the place value grid: Use of the part whole model: 	$48 \div 2 = 24$
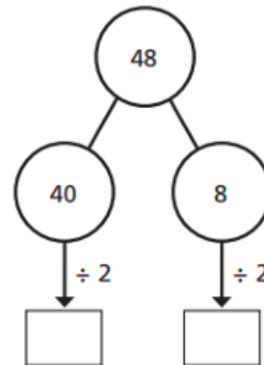
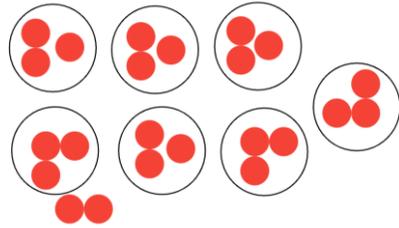| Year 4

Sharing 2 digit by 1 digit with exchange | |

$10 + 3 = 13$

52
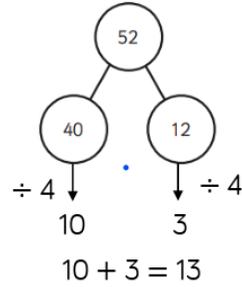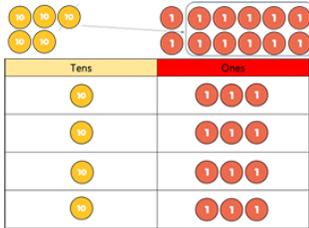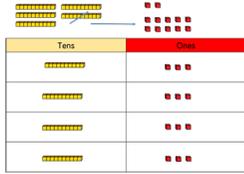
| ? | ? | ? | ? | | $52 \div 4 = 13$ |
| Sharing 2 digit by 1 digit with remainders | Use of counters to support understanding

23 divided by 3 (continue with the idea of sharing and grouping).

 | 53
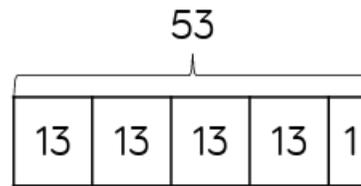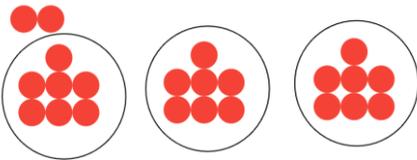
| 13 | 13 | 13 | 13 | 1 | | $53 \div 4 = 13\ r1$

Writing of multiplication facts alongside question to support. |

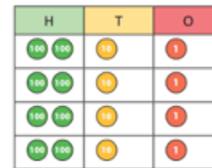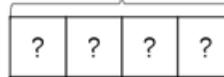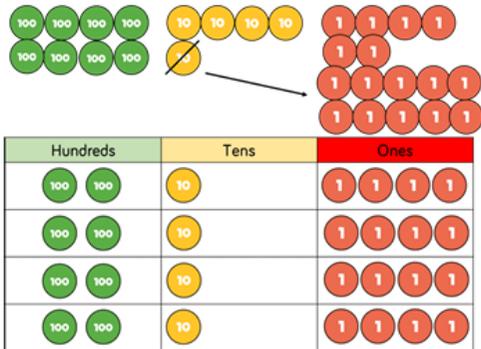| | Sharing 3 digits by 1 digit with no remainders and remainders | Without remainders:
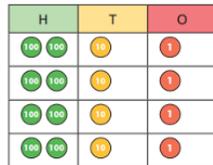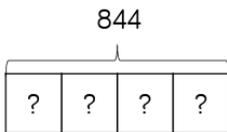
$$844 \div 4 = 211$$

With remainders: | | Without remainders:

$$844 \div 4 = 211$$ |

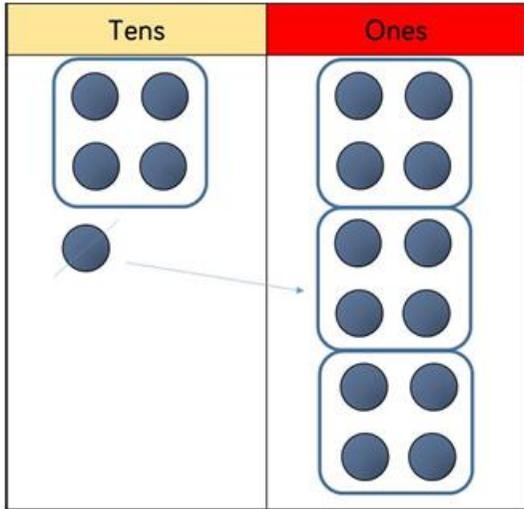| Objective	Concrete	Pictorial	Abstract
Year 5 2 digit by 1 digit with exchange using the formal written method			$52 \div 4 = 13$
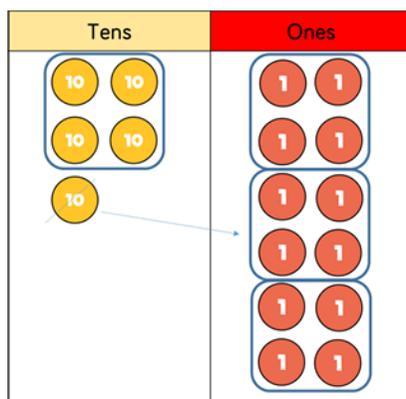
| Objective	Concrete	Pictorial	Abstract
3 or 4 digit by a 1 digit with exchange using the formal written method			Encourage the use of writing down their times tables to support.
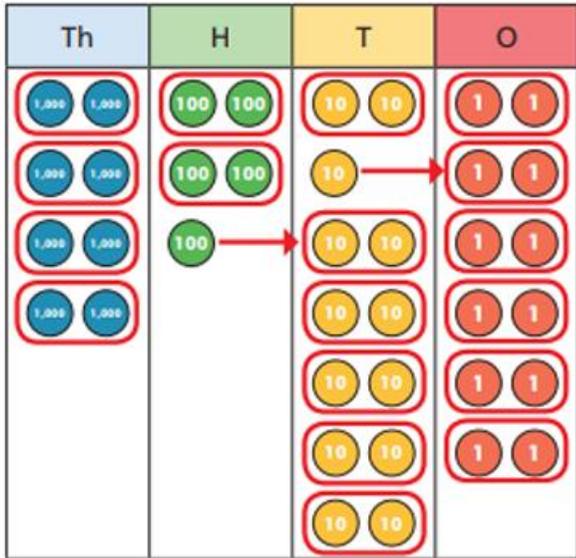
Year 6 Short division but with recognising remainders as decimals and fraction			86 r 2 (³) 5|432 Move onto divisions with a remainder. Once children understand remainders, begin to express as a fraction or decimal according to the context. 1 8 6 1/5 5|9 ⁴3 ³1 With the above, recognizing that one fifth is equivalent to two tenths so the answer is also 186.2 1 4 . 6 (16) (21) 3 5|5 1 1 . 0
Long division		Instead of using physical counters, children can draw the counters and circle the groups on a whiteboard or in their books. Use this method to explain what is happening and as soon as they have understood, move on to the abstract method as this can be time consuming.	Children will use long division to divide numbers with up to 4 digits by 2 digit numbers. The use of help box to support: 4567 divided by 26 Help Box: 10 x 26 = 260 5 x 26 = 130
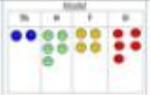
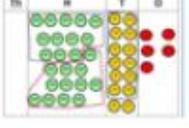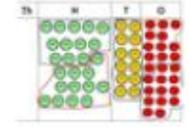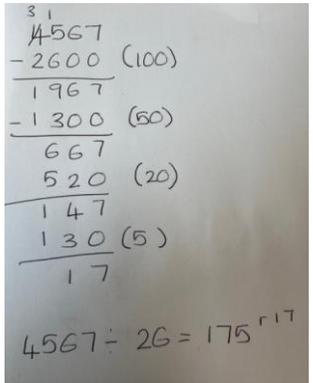|			
	  2544 ÷ 12 How many groups of 12 thousands do we have? None Exchange 2 thousand for 20 hundreds. How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one. Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2 Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	*Use of factor trees to support division:*		*(recognizing the relationship to above)* 2 x 26 = 52 1 x 26 = 26 *Use their knowledge of multiplying by 10 and 100 and that division is repeated subtraction:* *Or use of factors (factor tree that would have been taught previously)*

25
5 5
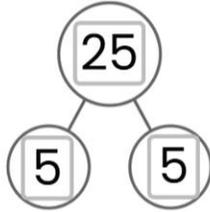
4565 divided by 5 and then divide by 5 again.

26
2 13
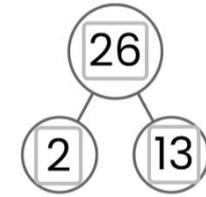
4567 divided by 2 = 2283.5

2283.5 divided by 13 = 175 r 17